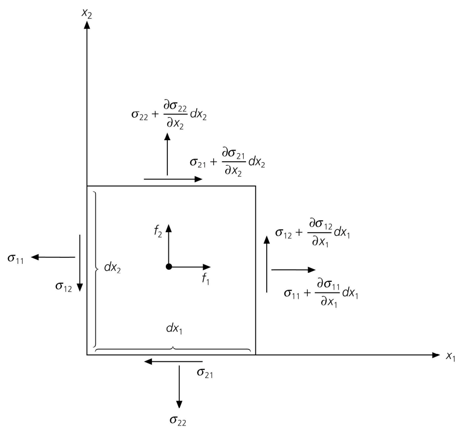
Seismic wave equation

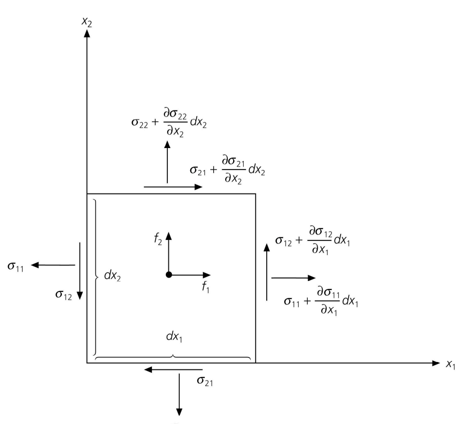


$$mass \times acceleration = \sum forces$$

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ij,j} + f_i$$

$$\rho \frac{\partial^2 u_i}{\partial t^2} = c_{ijkl} u_{(k,l),j} + f_i$$

Seismic wave equation



$$\rho \frac{\partial^2 u_i}{\partial t^2} = c_{ijkl} u_{(k,l),j} + f_i$$

isotropic medium:

$$c_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{il} \delta_{jk} + \delta_{ik} \delta_{jl})$$

Helmholtz decomposition:

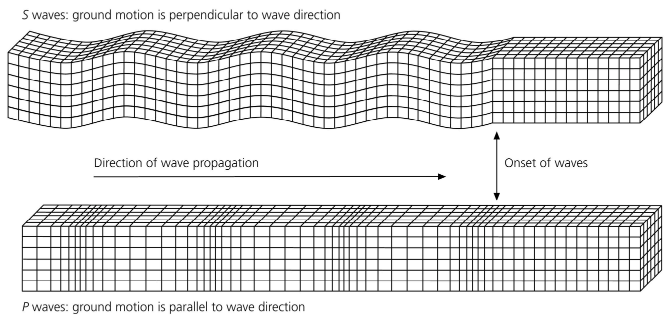
$$\underline{u} = \underline{\nabla} \phi + \underline{\nabla} \times \underline{\psi}$$

$$\underline{\nabla} \times \underline{\nabla} \phi = 0$$

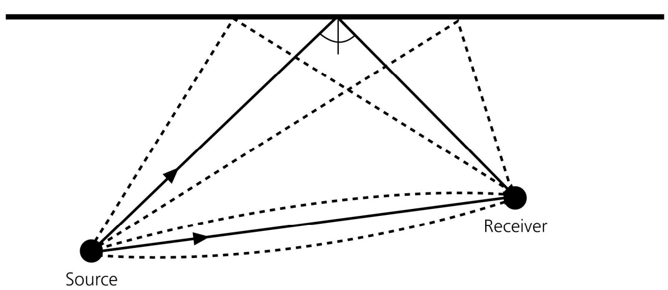
$$\underline{\nabla} \cdot \underline{\nabla} \times \underline{\psi} = 0$$

Seismic wave equation

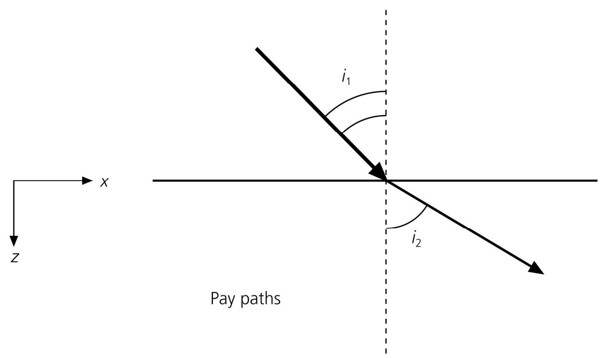
	P-wave	S-wave
wave eq:	$\alpha^2 \underline{\nabla}^2 \phi - \frac{\partial^2 \phi}{\partial t^2} = -\frac{1}{\rho} F_P$	$\beta^2 \underline{\nabla}^2 \underline{\psi} - \frac{\partial^2 \underline{\psi}}{\partial t^2} = -\frac{1}{\rho} \underline{F_S}$
velocity:	$\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}}$	$\beta = \sqrt{\frac{\mu}{\rho}}$



Ray theory

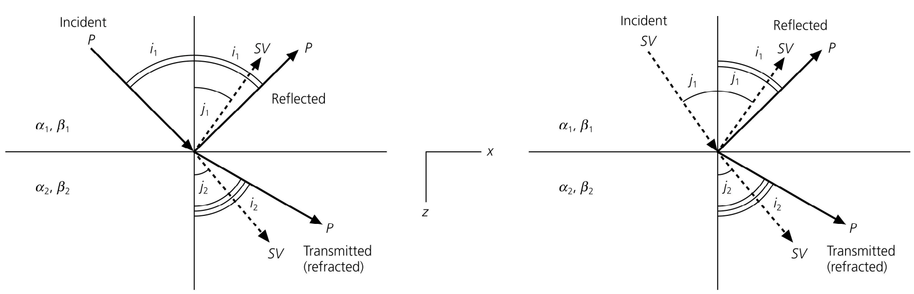


Snell's law



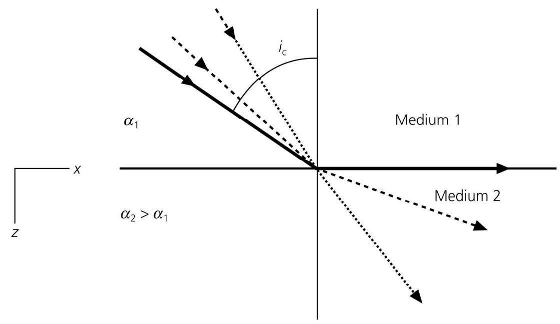
$$p = \frac{\sin i_1}{v_1} = \frac{\sin i_2}{v_2}$$

Snell's law



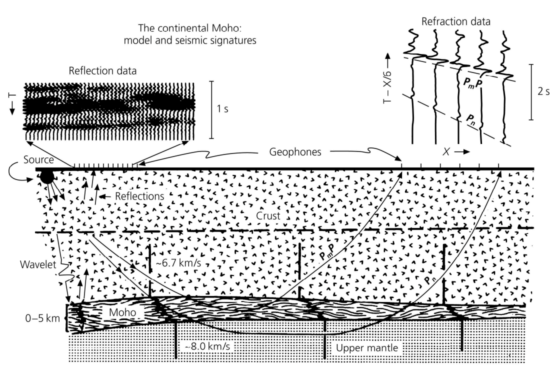
$$p = \frac{\sin i_1}{\alpha_1} = \frac{\sin j_1}{\beta_1} = \frac{\sin i_2}{\alpha_2} = \frac{\sin j_2}{\beta_2}$$

Head waves

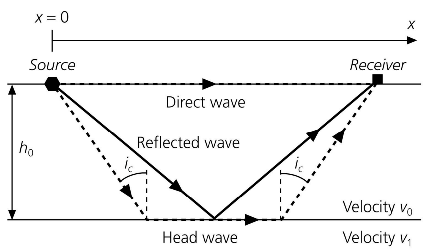


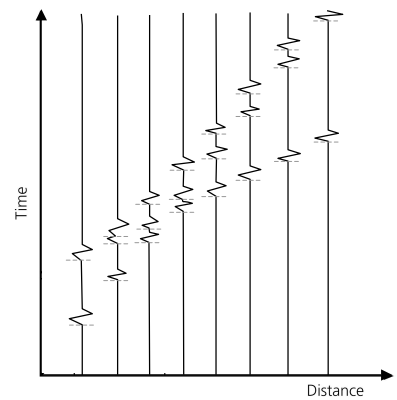
$$p_c = \frac{\sin i_c}{\alpha_1} = \frac{\sin(\pi/2)}{\alpha_2} = \frac{1}{\alpha_2}$$

Seismic reflection and refraction

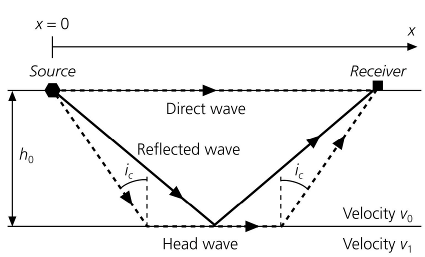


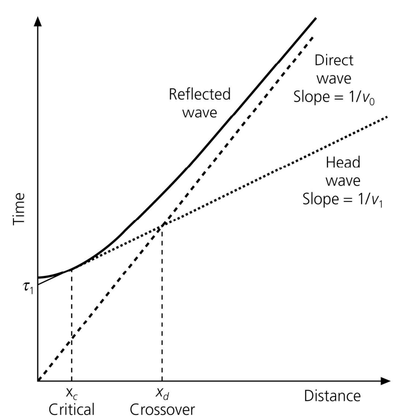
Seismic refraction



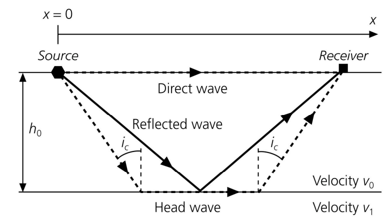


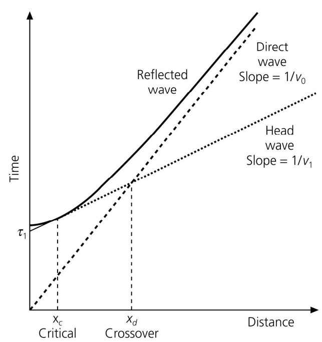
Seismic refraction





Seismic refraction





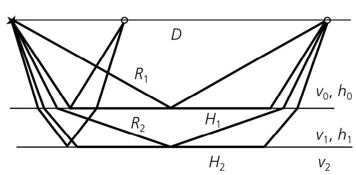
$$T_D(x) = \frac{x}{v_0}$$

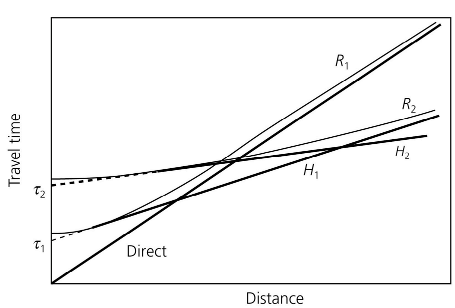
$$T_R(x) = 2\frac{\left(x^2/4 + h_0^2\right)^{1/2}}{v_0}$$
$$= px + 2\eta_0 h_0$$

$$T_H(x) = \frac{x}{v_1} + 2h_0\left(\frac{1}{v_0^2} - \frac{1}{v_1^2}\right)^{1/2}$$
$$= \frac{x}{v_1} + \tau_1$$

where vertical slownes is: $\eta_0 = \left(1/v_0^2 - p^2\right)^{1/2}$

Seismic refraction





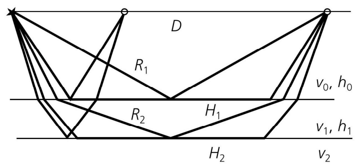
$$T_D(x) = \frac{x}{v_0}$$

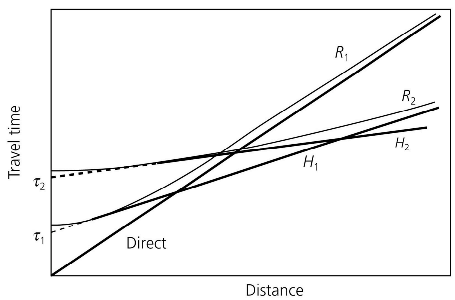
$$T_{Rn}(x) = px + 2\sum_{j=0}^{n-1} \eta_j h_j$$

$$T_{Hn}(x) = \frac{x}{v_n} + 2\sum_{j=0}^{n-1} h_j \left(\frac{1}{v_j^2} - \frac{1}{v_n^2}\right)^{1/2}$$
$$= \frac{x}{v_n} + \tau_n$$

$$h_{n-1} = \frac{\tau_n - 2\sum_{j=0}^{n-2} h_j \left(\frac{1}{v_j^2} - \frac{1}{v_n^2}\right)^{1/2}}{2\left(\frac{1}{v_{n-1}^2} - \frac{1}{v_n^2}\right)^{1/2}}$$

Seismic refraction

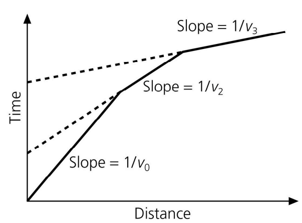


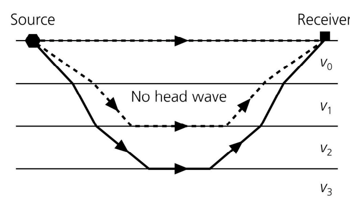


$$h_{n-1} = \frac{\tau_n - 2\sum_{j=0}^{n-2} h_j \left(\frac{1}{v_j^2} - \frac{1}{v_n^2} \right)^{1/2}}{2\left(\frac{1}{v_{n-1}^2} - \frac{1}{v_n^2} \right)^{1/2}}$$

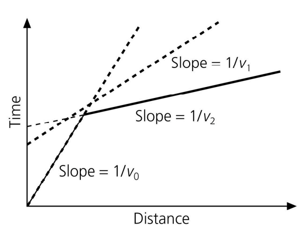
Seismic refraction

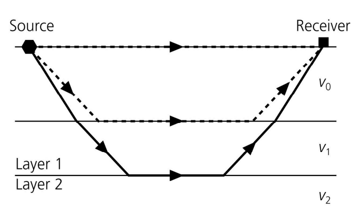
Low-velocity layer:



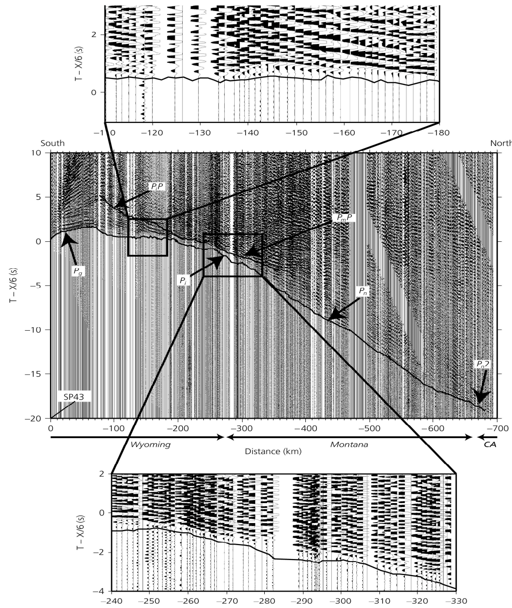


Blind zone:

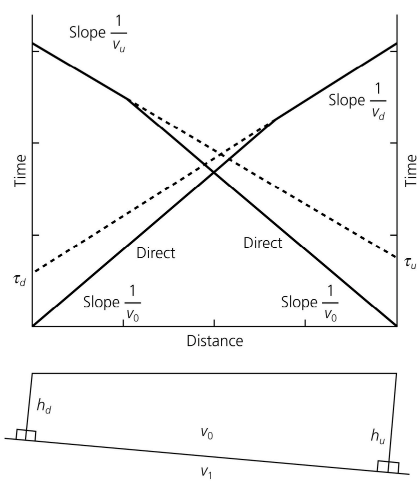




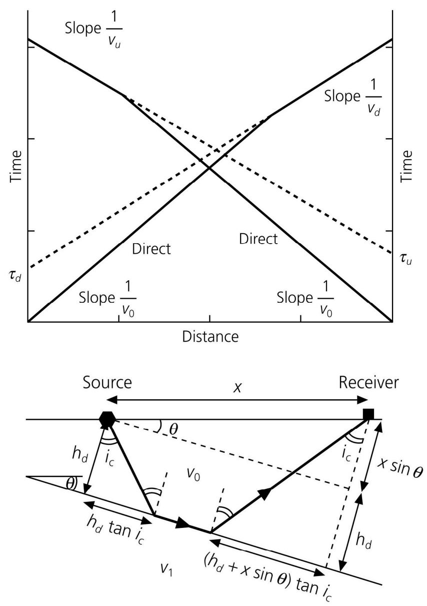
Seismic refraction



Seismic refraction



Seismic refraction



$$T_d(x) = \frac{x\cos\theta - (2h_d + x\sin\theta)\tan i_c}{v_1} + \frac{(2h_d + x\sin\theta)}{v_0\cos i_c}$$

$$= \frac{x\sin(i_c+\theta)}{v_0} + \frac{2h_d\cos i_c}{v_0}$$

$$= \frac{x}{v_d} + \tau_d$$

$$T_u(x) = \frac{x\cos\theta - (2h_u - x\sin\theta)\tan i_c}{v_1} + \frac{(2h_u - x\sin\theta)}{v_0\cos i_c}$$

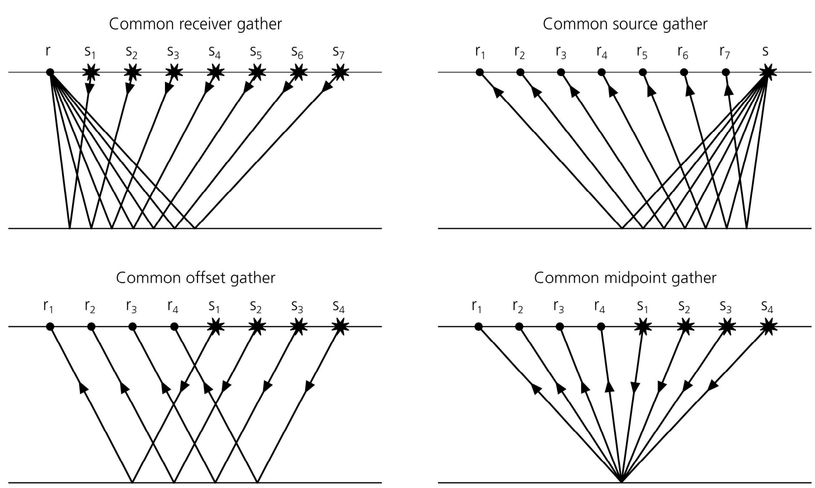
$$= \frac{x\sin(i_c-\theta)}{v_0} + \frac{2h_u\cos i_c}{v_0}$$

$$= \frac{x}{v_u} + \tau_u$$

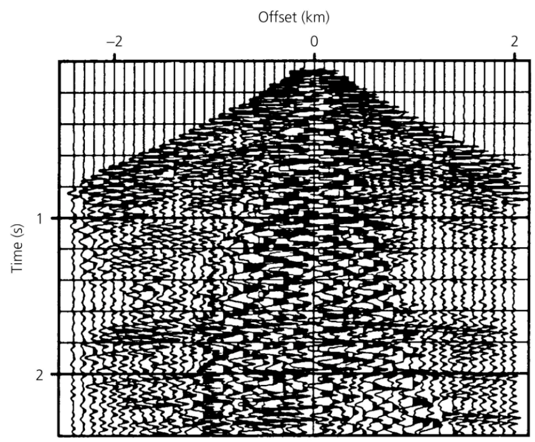
$$\theta = \frac{1}{2}\left(\sin^{-1}\frac{v_0}{v_d} - \sin^{-1}\frac{v_0}{v_u}\right)$$

$$i_c = \frac{1}{2}\left(\sin^{-1}\frac{v_0}{v_d} + \sin^{-1}\frac{v_0}{v_u}\right)$$

Seismic reflection

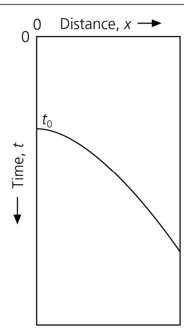


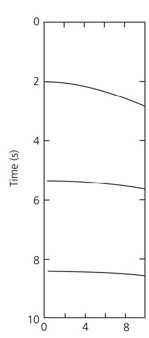
Seismic reflection

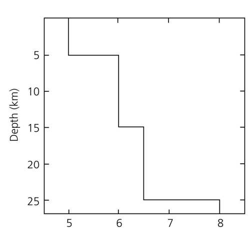


CMP gather

Seismic reflection







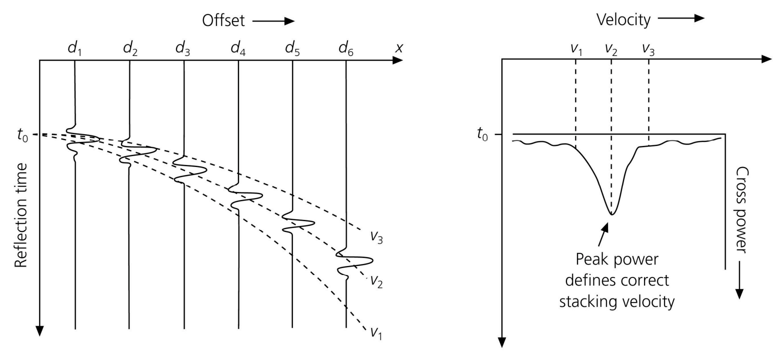
Normal moveout (NMO) correction:

$$\Delta T_{NMOn}(x) = T_{Rn}(x) - t_n$$

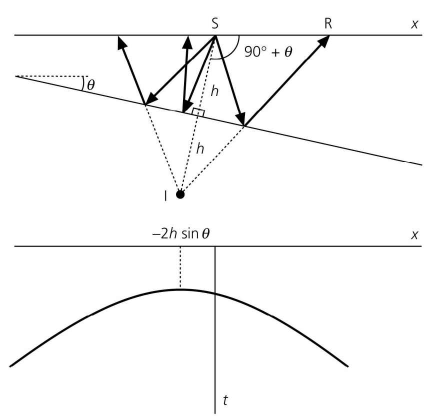
$$= px + 2\sum_{j=0}^{n-1} \eta_j h_j - t_n$$

$$t_n = 2\sum_{j=0}^{n} h_j / v_i$$

Seismic reflection



Seismic reflection



$$T_R(x) = \frac{\left(x^2 + 4h^2 + 4hx\sin\theta\right)}{v_0^2}$$

Seismic reflection

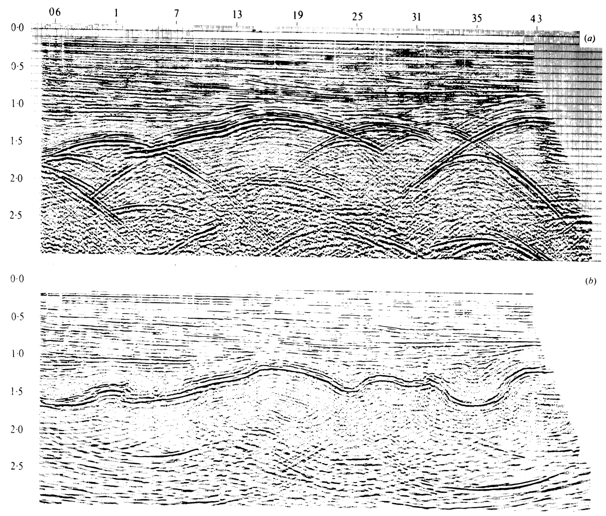


Fig. 9.57 Migration removes the confusion produced by multiple-branch reflections. The deeper data, which are probably multiples, out-of-plane diffractions, and other types of noise are smeared out because they are migrated as if they were primary reflections. (Courtesy of AGIP.) (a) Unmigrated section; (b) migrated.

Sheriff and Geldart, 1994